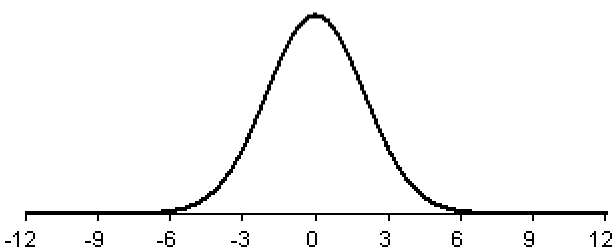
ESTIMATING VARIANCE

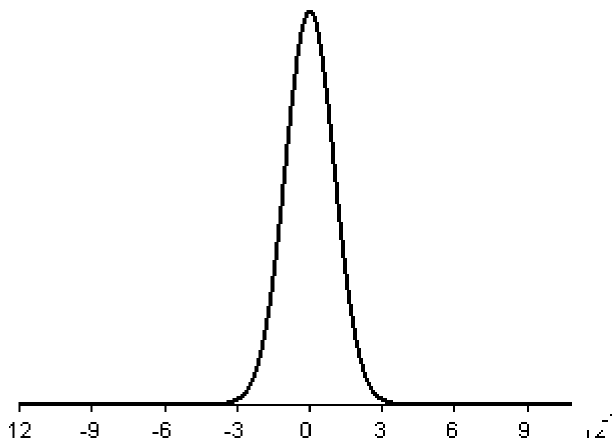
The three curves below represent standard deviations of 1, 2 and 3.

Which curve below would represent a standard deviation of 1? How do you know?

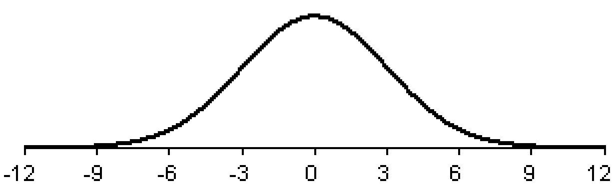
Which curve would represent a standard deviation of 3? How do you know?



A



B



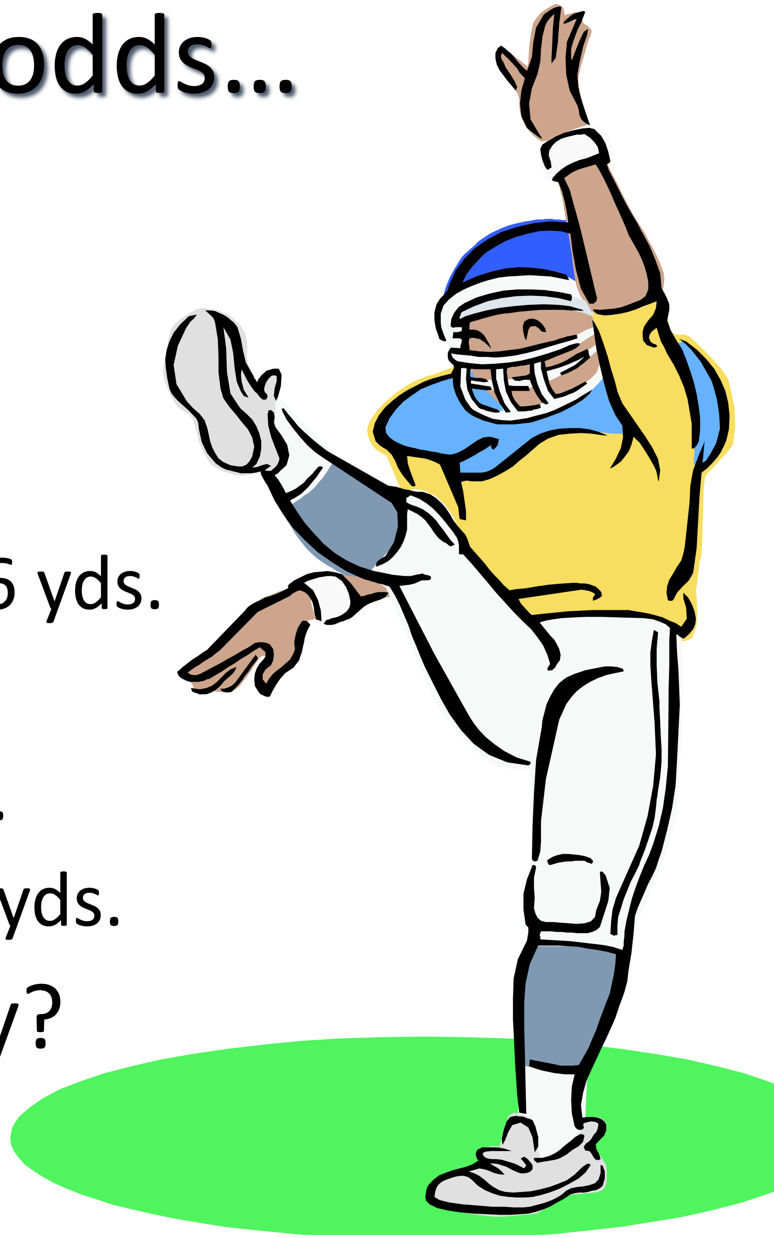
C

THE GREATER THE VARIANCE IN RESULTS, THE GREATER THE STANDARD DEVIATION.

[Standard deviation, the normal curve and baseball.]

Weighing the odds…

- 2 High School punters
  - Kicker A:
    - mean distance: 40.0 yds
    - Standard deviation: $\pm$ 16 yds.
  - Kicker B:
    - mean distance: 34.5 yds.
    - Standard deviation: $\pm$ 4 yds.
- Which player do you play?
  - Kicker B – team will know what to expect

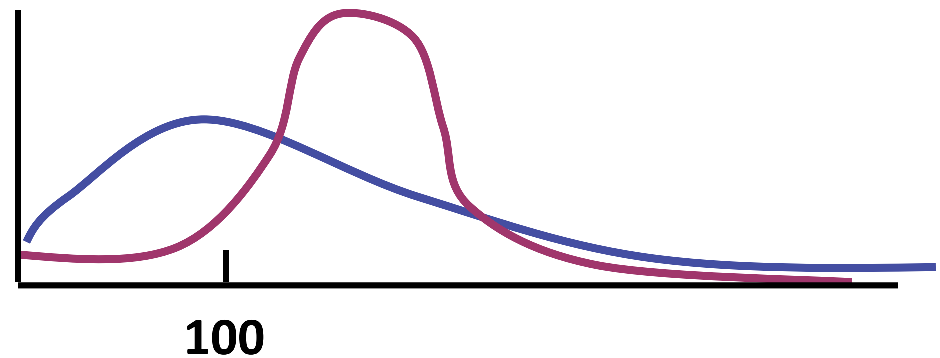


Applying the concepts

Try, with the help of this rough drawing below, to describe intelligence test scores at a high school and at a college using the concepts of range and standard deviation.

Intelligence test scores at a high school

Intelligence test scores at a college



Want to take that class?

- So, if you were told that the mean average in a class was 85%, with a standard deviation of 5, would you feel confident that you would get a "B"?
- ***<u>The smaller the standard deviation, the more closely the scores are packed near the mean, and the steeper the curve would appear.</u>***
- What percentage of students got a B or A? 84%
- What would the standard deviation be if every score was the mean score? 0

Z-scores

- Sometimes being able to compare scores from different distributions is important.
- Z-scores measure distance of a score from the mean in units of standard deviation.
- Scores below the mean have negative z-scores, and those above have positive z-scores.
- FOR EXAMPLE: Test: Mean = 80 & SD = 8

  Phineas got a 72%: z-score of -1

  Ferb got a 84%: z-score of +0.5

Direction of a Z-score

- The sign of any Z-score indicates the direction of a score: whether that observation fell above the mean (the positive direction) or below the mean (the negative direction)
  - If a raw score is below the mean, the z-score will be negative, and vice versa

Comparing variables with very different observed units of measure

- Example of comparing an SAT score to an ACT score
  - Mary's ACT score is 26. Jason's SAT score is 900. Who did better?
  - The mean SAT score is 1000 with a standard deviation of 100 SAT points. The mean ACT score is 22 with a standard deviation of 2 ACT points.

Let's find the z-scores

$$Z = \frac{\text{Score-mean}}{\text{SD}}$$

Jason: $$Z_x = \frac{900-1000}{100} = -1$$

Mary: $$Z_x = \frac{26-22}{2} = +2$$

- From these findings, we gather that Jason's score is 1 standard deviation below the mean SAT score and Mary's score is 2 standard deviations above the mean ACT score.
- Therefore, Mary's score is relatively better.

Interpreting the graph

- For any normally distributed variable:
  - 50% of the scores fall above the mean and 50% fall below.
  - Approximately 68% of the scores fall within plus and minus 1 Z-score from the mean.
  - Approximately 95% of the scores fall within plus and minus 2 Z-scores from the mean.
  - 99.7% of the scores fall within plus and minus 3 Z-scores from the mean.

Z – Score Conclusions

- Z-score is defined as the number of standard deviations from the mean.
- Z-score is useful in <u>comparing variables with very different observed units of measure.</u>

    (Like measures of central tendency and variation – z-scores can <u>*describe.)*</u>

- HOWEVER -

- Z-scores allow for precise <u>***predictions***</u> to be made of how many of a population's scores fall within a score range in a normal distribution.

    (So they are also inferential, because they can infer what might happen in the future.)

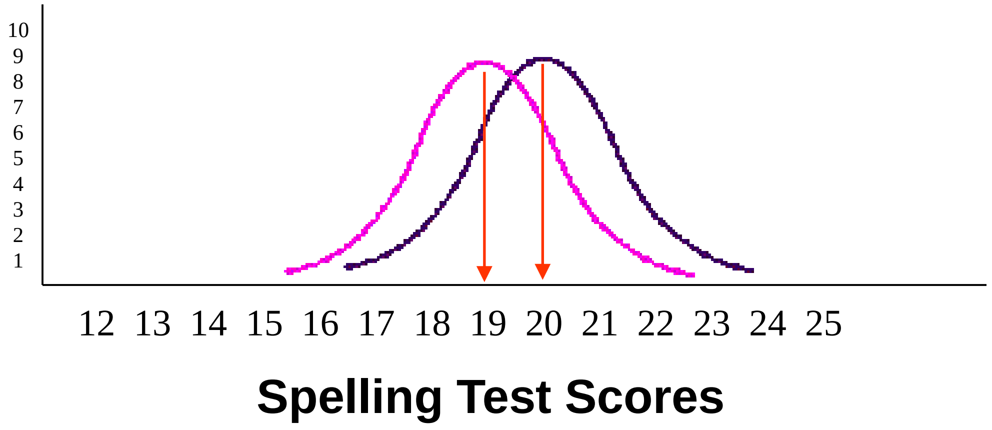
Types of statistics

- Descriptive statistics are used to *reveal patterns* through the analysis of numeric data.
  - Measures of Central tendency
  - Measures of variation
  - Z-scores

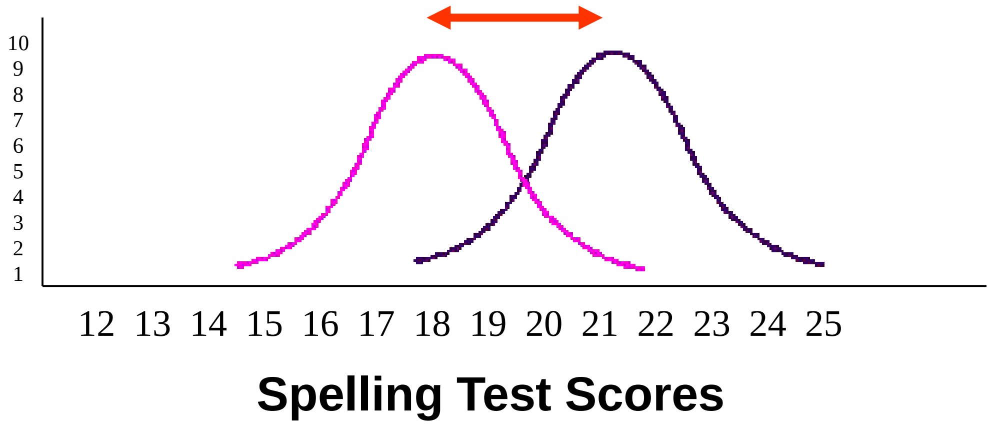
- Inferential statistics are used to *draw conclusions and make predictions* based on the analysis of numeric data.
  - Z-scores
  - t-tests
    - These types of stats help us determine if ***chance*** played a role in our findings.

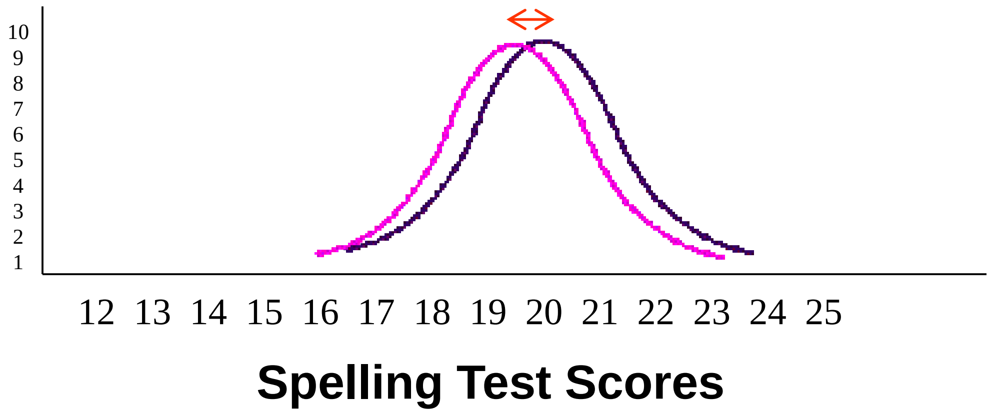
Drawing conclusions – did our experiment work?

- Suppose we conducted a study to compare two strategies for teaching spelling.
- Group A had a mean score of 19. The range of scores was 16 to 22, and the standard deviation was 1.5.
- Group B had a mean score of 20. The range of scores was 17 to 23, and the standard deviation was 1.5.
- How confident can we be that the difference we found between the means of Group A and Group B occurred because of differences in our reading strategies, rather than by chance?



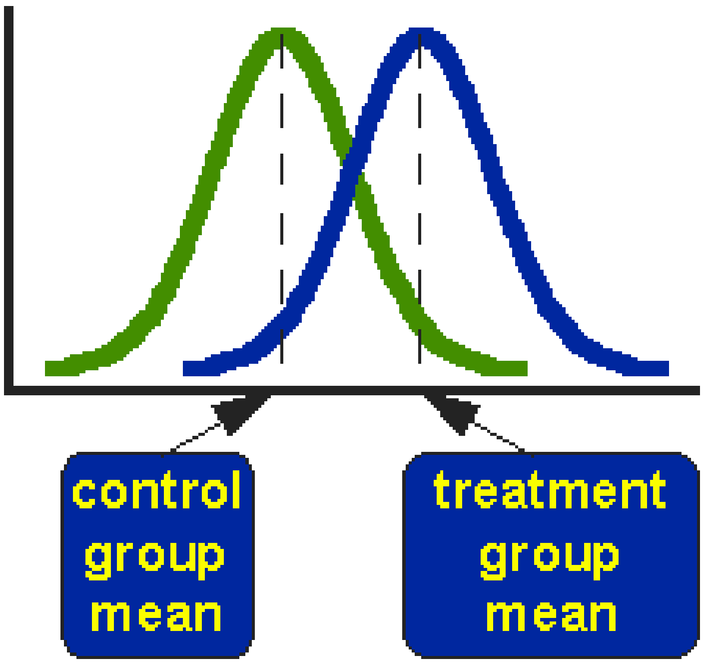
All other factors being equal, large differences between means are less likely to occur by chance than small differences.



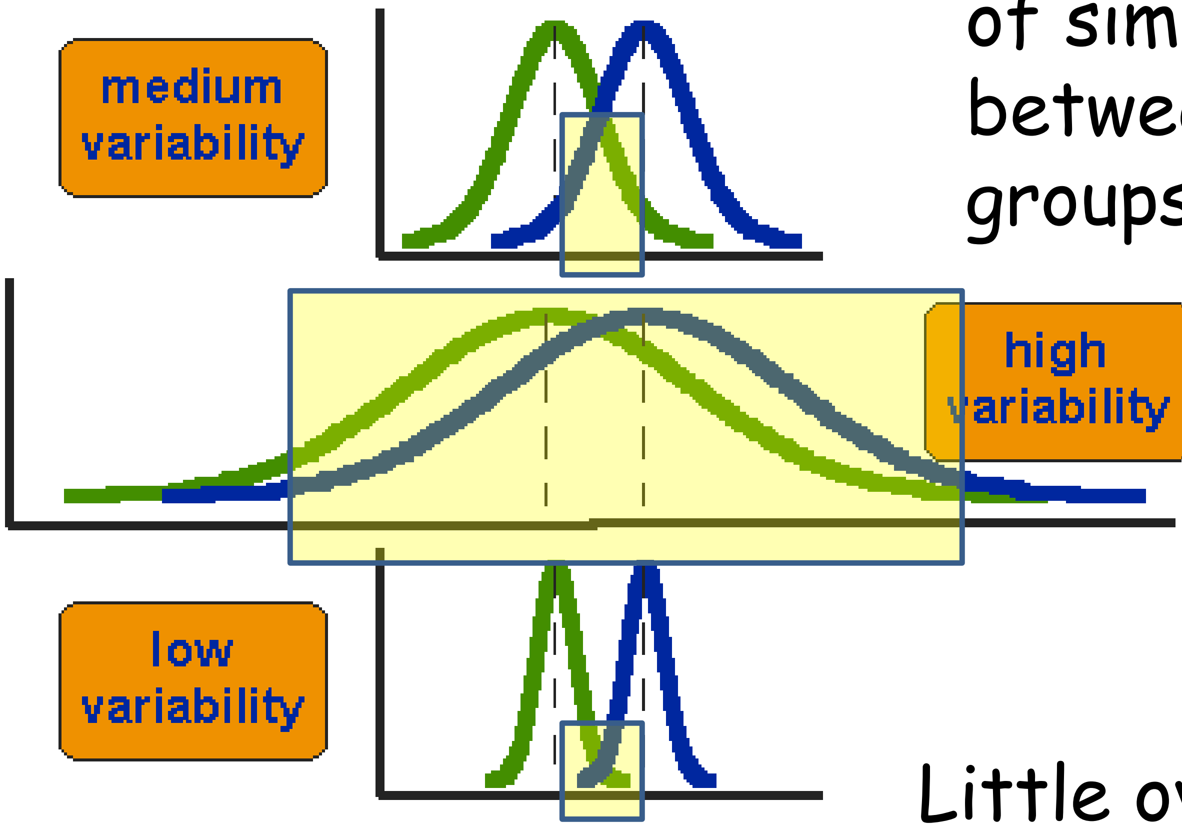


t-test

- ***In summary:***
- Assesses whether the means of two groups are *statistically* different from each other by comparing the means.
- This analysis is appropriate whenever you want to compare the means of two groups.



Large overlap, a lot of similarity between the two groups.



Little overlap, large differences between groups.

T-Tests are a good way to determine whether a finding is statistically significant

Inferential Statistics

When is a Difference Significant?

p-values

Statistically Significant: a result is called **statistically significant** if it is unlikely to have occurred by chance.

"Magic number" is $p \leq .05$

This means you are 95% sure the results did not occur by chance.

- The purpose is to discover whether the finding can be applied to the larger population from which the sample was collected.